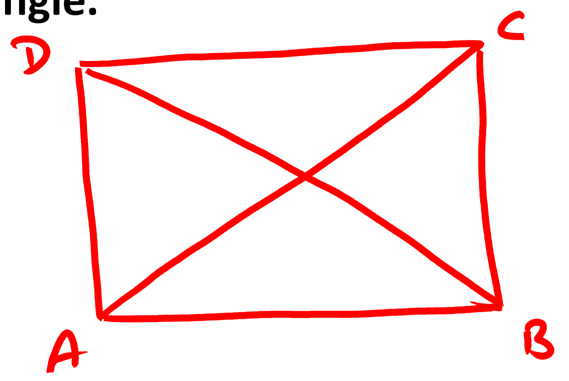
Exercise 8.1

1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.



Given: ABCD is a parallelogram

$AC = BD$

To prove: ABCD is a rectangle.

Proof: In $\triangle DAB$ & $\triangle CBA$

$DA = CB$ (opposite sides of ||gm are equal)

$AB = AB$ (common)

$DB = AC$ (Given)

$\therefore \triangle DAB \cong \triangle CBA$ by SSS rule

$\therefore \angle DAB = \angle CBA$ (cpct)

Now, $\angle DAB + \angle CBA = 180^\circ$ (Co-interior angles)

or (Adjacent $\angle$s of ||gm are supplementary)

Exercise 8.1

$\angle DAB + \angle DAB = 180^\circ \quad (\because \angle DCB = \angle DAB)$

$2\angle DAB = 180^\circ$

$\angle DAB = \frac{\cancel{180}}{\cancel{2}} 90^\circ$

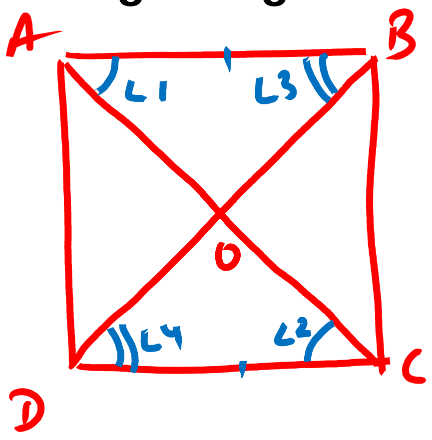
As one angle of ||gm ABCD is 90°,

$\therefore$ ABCD is a rectangle.

Hence proved.

Exercise 8.1

2. Show that the diagonals of a square are equal and bisect each other at right angles.



Given: ABCD is a square

To prove: (i) $AC = BD$ (ii) $AO = OC$, $BO = OD$ (iii) $AO \perp BD$

Proof: In $\triangle ADC$ and $\triangle BCD$

$AD = BC$ (All sides of square are equal)

$\angle ADC = \angle BCD$ (Each angle $= 90°$)

$DC = DC$ (common)

$\therefore \triangle ADC \cong \triangle BCD$ by SAS rule

$\therefore \boxed{AC = BD}$ (CPCT)

In $\triangle AOB$ and $\triangle COD$

$\angle 1 = \angle 2$ (alternate interior angles)

$AB = CD$ (All sides of square are equal)

$\angle 3 = \angle 4$ (alternate interior angles)

Exercise 8.1

$\therefore \triangle AOB \cong \triangle COD$ by ASA rule

$\therefore$ $\boxed{\begin{aligned} AO &= CO \\ \text{and } OB &= OD \end{aligned}}$ (cpct) (cpct)

In $\triangle AOD$ and $\triangle AOB$

$AD = AB$ (All sides of square are equal)

$DO = OB$ (proved above)

$AO = AO$ (common)

$\therefore \triangle AOD \cong \triangle AOB$ by SSS rule

$\angle AOD = \angle AOB$ (cpct)

$\angle AOD + \angle AOB = 180^\circ$ (linear pair)

$\angle AOD + \angle AOD = 180^\circ$ ($\because \angle AOB = \angle AOD$)

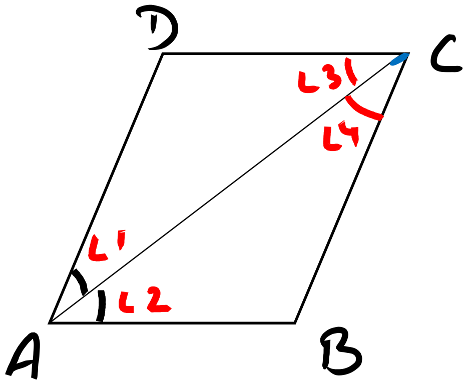
$2\angle AOD = 180^\circ$

$\angle AOD = \cancel{\frac{180}{2}}\ 90^\circ$

$\therefore \boxed{AO \perp BD}$ Hence proved

Exercise 8.1

3. Diagonal AC of a parallelogram ABCD bisects ∠ A (see Fig. 8.11). Show that
(i) it bisects ∠ C also,
(ii) ABCD is a rhombus.



(fig 8.11)

Given: ABCD is a parallelogram

$\angle 1 = \angle 2$

To prove: 1) $\angle 3 = \angle 4$ 2) ABCD is a rhombus

Proof: As ABCD is a ||gm

$\therefore$ AD || BC

and AB || DC (opposite sides of ||gm are ||)

$\angle 1 = \angle 2$ (Given)

$\angle 1 = \angle 4$ (alternate interior ∠s)

$\therefore \angle 2 = \angle 4$

Also $\angle 2 = \angle 3$ (alternate interior ∠s)

$\therefore \boxed{\angle 3 = \angle 4}$

Exercise 8.1

In $\triangle ABC$,

$\angle 2 = \angle 4$ (proved above)

$\therefore BC = AB$ (sides opposite to equal angles are equal in length)

As adjacent sides of parallelogram ABCD are equal

$\therefore$ $\boxed{\text{ABCD is a rhombus}}$

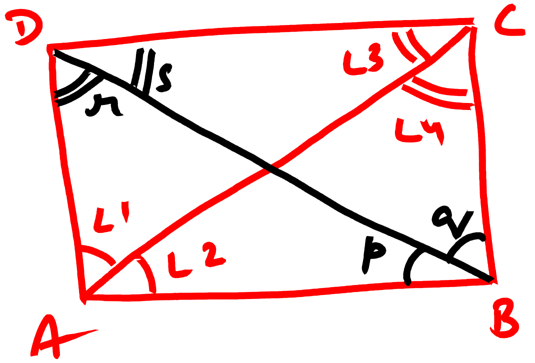
Hence proved.

Exercise 8.1

4. ABCD is a rectangle in which diagonal AC bisects ∠ A as well as ∠ C. Show that:

(i) ABCD is a square

(ii) diagonal BD bisects ∠ B as well as ∠ D.



Given: ABCD is a rectangle

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

To prove: (i) ABCD is a square

(ii) $p = q$ and $r = s$

Proof: As ABCD is a rectangle

$\therefore$ AD || BC and AB || DC (opposite sides of rectangle are ||)

$\therefore \angle 1 = \angle 4$ (alternate interior ∠s)

Also $\angle 1 = \angle 2$ (given)

$\therefore \angle 4 = \angle 2$

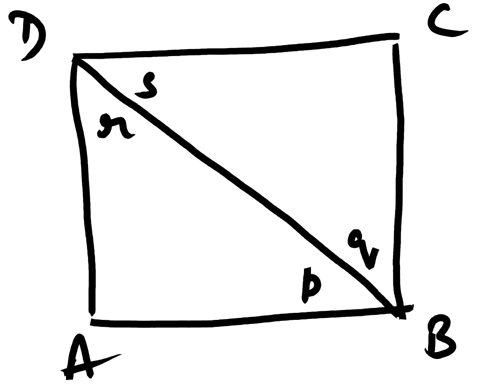
In $\triangle ABC$,

$\angle 4 = \angle 2$ (proved above)

Exercise 8.1

$\therefore AB = BC$ (sides opposite to equal angles are equal)

As adjacent sides of rectangle ABCD are equal, $\therefore$ ABCD is a square.



(ii) In $\triangle ABD$ and $\triangle CBD$

$AB = CB$ (all sides of sq are equal)

$AD = CD$ (all sides of sq are equal)

$BD = BD$ (common)

$\therefore \triangle ABD \cong \triangle CBD$ by SSS rule

$\therefore p = q$ (cpct)

and $r = s$ (cpct)

or, BD bisects $\angle B$ as well as $\angle D$.

Hence proved.

Exercise 8.1

5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.12). Show that:

(i) Δ APD $\cong$ Δ CQB

(ii) AP = CQ

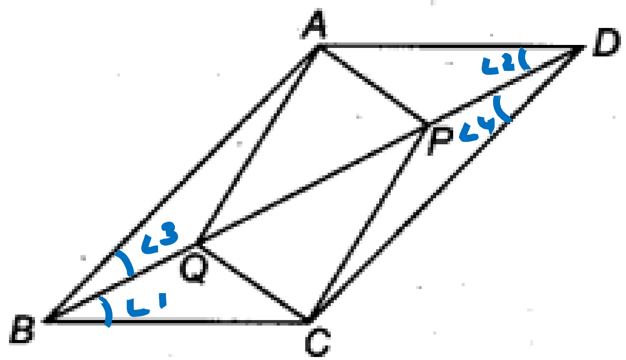
(iii) Δ AQB $\cong$ Δ CPD

(iv) AQ = CP

(v) APCQ is a parallelogram

Given: ABCD is a ||gm
DP = BQ.

To prove:



Proof: In ΔAPD and ΔCQB

AD = BC (opposite sides of ||gm are equal)

∠2 = ∠1 (alternate interior angles)

DP = BQ (given)

∴ $\boxed{\Delta APD \cong \Delta CQB}$ by SAS rule

∴ $\boxed{AP = CQ}$ (cpct)

In ΔAQB and ΔCPD

AB = CD (opposite sides of ||gm are equal)

∠3 = ∠4 (alternate interior ∠s

Exercise 8.1

$BQ = PD$ (given)

$\therefore \boxed{\Delta AQB \cong \Delta CPD}$ by SAS rule

$\therefore \boxed{AQ = CP}$ (cpct)

In $\square APCQ$,

$AQ = CP$

and $AP = CQ$

Both pairs of opposite sides of $\square APCQ$ are equal

$\therefore$ APCQ is a parallelogram.

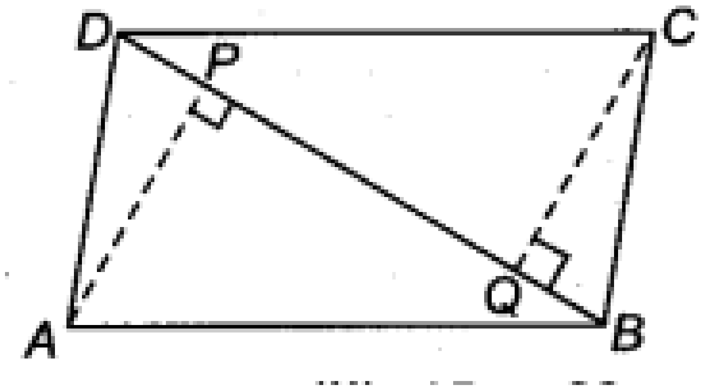
Hence proved

Exercise 8.1

6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.13). Show that

(i) Δ APB $\cong$ Δ CQD

(ii) AP = CQ



Given : ABCD is a parallelogram

AP $\perp$ BD and CQ $\perp$ BD

To prove : i) $\Delta APQ \cong \Delta CQD$

ii) AP = CQ

Proof : In ΔAPB and ΔCQD

AB = CD (opposite sides of ||gm are equal)

$\angle APB = \angle CQD$ (Each 90°)

$\angle ABP = \angle CDQ$ (alternate interior angles)

$\therefore \Delta APB \cong \Delta CQD$ by AAS rule

$\therefore$ AP = CQ (cpct)

Exercise 8.1

7. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.14). Show that *

Given: $AB \parallel CD$, $AD = BC$

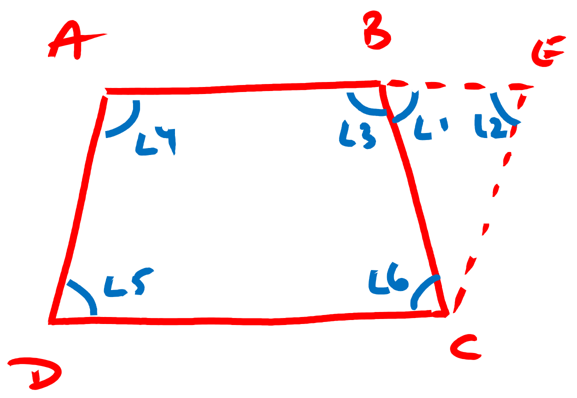
To prove:

(i) ∠ A = ∠ B

(ii) ∠ C = ∠ D

(iii) Δ ABC ≅ Δ BAD

(iv) diagonal AC = diagonal BD



Construction: Through C, draw a line parallel to DA, intersecting AB produced at E.

Proof: As $AB \parallel CD$ and $AD \parallel CE$

$\therefore$ ADCE is a parallelogram

$\therefore AD = CE$ (opp sides of ||gm are =)

Also $AD = BC$ (given)

$\therefore CE = BC$

In Δ CEB,

$CE = CB$ (proved above)

$\therefore \angle 1 = \angle 2$ (angles opp to = sides are =)

$\angle 1 + \angle 3 = 180^\circ$ (linear pair)

$\angle 2 + \angle 4 = 180^\circ$ (co interior ∠s)

$\therefore \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle 1 + \angle 3 - \angle 2 = \angle 4$

$\cancel{\angle 1} + \angle 3 - \cancel{\angle 1} = \angle 4$ ($\because \angle 2 = \angle 1$)

$\angle 3 = \angle 4$

or $\boxed{\angle B = \angle A}$

Exercise 8.1

$\angle 4 + \angle 5 = 180^\circ$ (co-interior angles)

$\angle 3 + \angle 6 = 180^\circ$ (co-interior angles)

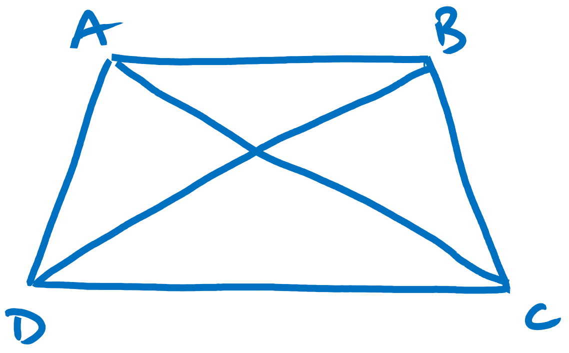
$\therefore \angle 4 + \angle 5 = \angle 3 + \angle 6$

$\angle 4 + \angle 5 - \angle 3 = \angle 6$

$\cancel{\angle 4} + \angle 5 - \cancel{\angle 4} = \angle 6 \quad (\because \angle 3 = \angle 4)$

$\angle 5 = \angle 6$

or $\boxed{\angle D = \angle C}$



In $\triangle ABC$ and $\triangle BAD$

$AB = AB$ (common)

$\angle ABC = \angle BAD$ (proved above)

$AD = BC$ (Given)

$\therefore \boxed{\triangle ABC \cong \triangle BAD}$ by SAS

$\therefore \boxed{AC = BD}$ (cpct)

Hence proved