Questions

1. ABCD is a quadrilateral in which P, Q, R and S are mid-pointsofthesides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal.

Show that :

(i) SR || AC and SR = AC/2

(ii) PQ = SR

(iii) PQRS is a parallelogram.

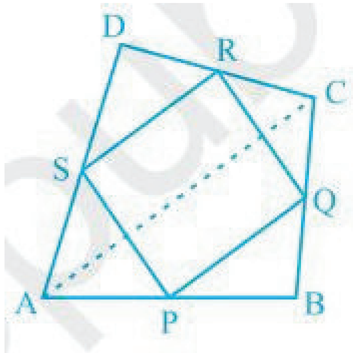


Fig. 8.29

Given: P, Q, R and S are mid-points of AB, BC, CD and DA.

To prove: —— *

Proof: In $\triangle DBC$,

S is mid point of AD

R is mid point of CD

$\therefore$ by mid point theorem

$SR \parallel AC$ and $SR = \frac{1}{2}AC$ — ①

In $\triangle ABC$

P is mid point of AB

Q is mid point of BC

$\therefore$ by mid point theorem

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ — ②

from ① and ②

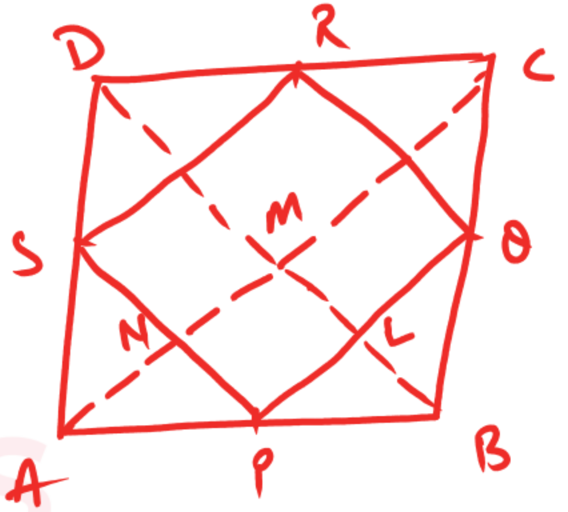
$PQ = SR$ and $PQ \parallel SR$

As one pair of opposite sides of $\square PQRS$ is equal and parallel

$\therefore$ PQRS is a parallelogram.

Questions

2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.



Given: ABCD is a rhombus. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively.

To prove: PQRS is a rectangle.

Constn: Draw AC and BD

Proof: In $\triangle ABC$

P is mid-point of AB

Q is mid-point of BC

$\therefore$ by mid point theorem

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ — (1)

In $\triangle DBC$

R is mid point of CD

S is mid point of AD

$\therefore$ by mid point theorem

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ — (2)

from (1) and (2)

$SR \parallel PQ$ and $SR = PQ$

As one pair of opposite sides is equal and parallel

$\therefore$ PQRS is a parallelogram

Questions

In $\triangle ABD$

P is mid point of AB

S is mid point of AD

$\therefore$ by mid point theorem

$PS \parallel BD$

As $PS \parallel BD$ and $PQ \parallel AC$

$\therefore$ $PLMN$ is a $\parallel$gm

$\therefore \angle LPN = \angle LMN$ (opp. $\angle$s of $\parallel$gm are equal)

$\angle LMN = 90°$ (Diagonals of a rhombus bisect at $90°$)

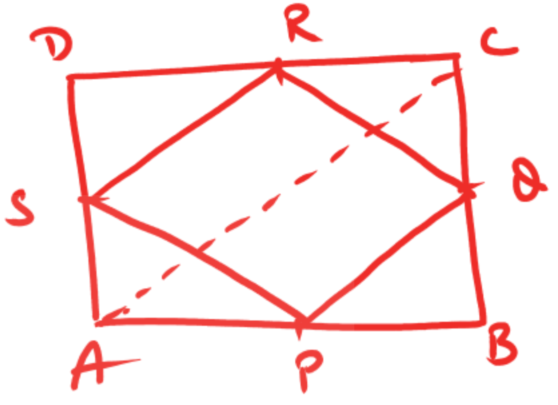
$\therefore \angle LPN = 90°$

As one angle of Parallelogram $PQRS$ is $90°$

$\therefore$ $PQRS$ is a rectangle.

Questions

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB,BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus



Given: ABCD is a rectangle. P, Q, R and S are mid-points of AB, BC, CD and DA respectively.

To prove: PQRS is a rhombus.

Constn: Join AC

Proof: In $\triangle ABC$,

P is mid point of AB

Q is mid point of BC

$\therefore$ by mid point theorem

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ — ①

In $\triangle ADC$,

S is mid point of AD

R is mid point of CD

$\therefore$ by mid point theorem

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ — ②

from ① and ②

$PQ \parallel SR$ and $PQ = SR$.

As one pair of opposite sides of $\square PQRS$ is equal and parallel

$\therefore$ PQRS is a parallelogram.

S is mid point of AD

$\therefore AS = SD = \frac{1}{2} AD$

Similarly $BQ = QC = \frac{1}{2} BC$

Questions

$AD = BC$ (opp. sides of rectangle are equal)

$\frac{1}{2}AD = \frac{1}{2}BC$ (halves of equals are equal)

$\therefore AS = BQ$

In ΔAPS and ΔBPQ

$AS = BQ$ (proved above)

$\angle SAP = \angle QBP$ (each 90°)

$AP = PB$ ($\because$ P is mid point of AB)

$\therefore \Delta APS \cong \Delta BPQ$ by SAS rule

$\therefore PS = PQ$ (cpct)

As adjacent sides of parallelogram PQRS are equal

$\therefore$ PQRS is a rhombus.

Questions

4. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.30). Show that F is the mid-point of BC.

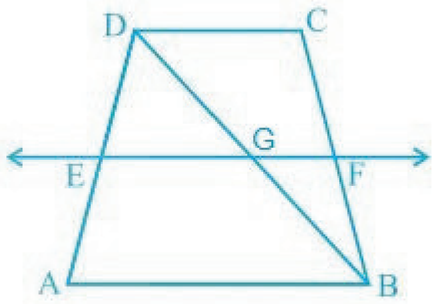


Fig. 8.30

Given : $AB \parallel DC$

$EF \parallel AB$

E is mid point of AD.

To prove : F is mid point of BC.

Proof : As $AB \parallel DC$

and $EF \parallel AB$

$\therefore AB \parallel DC \parallel EF$

In $\triangle DAB$,

E is mid point of AD

$EG \parallel AB$

$\therefore$ by converse of mid point theorem

G is mid point of DB.

In $\triangle BCD$,

G is mid point of BD

and $GF \parallel DC$

$\therefore$ by converse of mid point theorem

F is mid point of BC

Hence proved

Questions

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.22). Show that the line segments AF and EC trisect the diagonal BD.

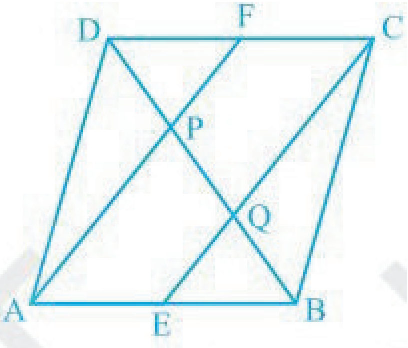


Fig. 8.31

Given: ABCD is a parallelogram
E is mid point of AB
F is mid point of CD

To prove: AF and EC trisect BD
i.e $DP = PQ = QB$

Proof: $AB \parallel CD$
$\therefore AE \parallel CF$

$AB = CD$ (opp sides of ||gm are =)

$\frac{1}{2}AB = \frac{1}{2}CD$ (halves of equals are =)

$\therefore AE = CF$ ($\because$ E is m.p of AB and F is m.p of CD)

As one pair of opposite sides of AECF is equal and parallel

$\therefore$ AECF is a parallelogram

In ΔDQC, F is mid point of DC and $PF \parallel CQ$
$\therefore$ by converse of mid point theorem P is mid point of DQ
i.e $DP = PQ$ — ①

In ΔABP, E is mid point of AB and $EQ \parallel AP$
$\therefore$ by converse of mid-point theorem Q is mid point of BP
i.e $PQ = QB$ — ②

Questions

from ① and ②

$\boxed{DP = PQ = QB}$

Hence proved.

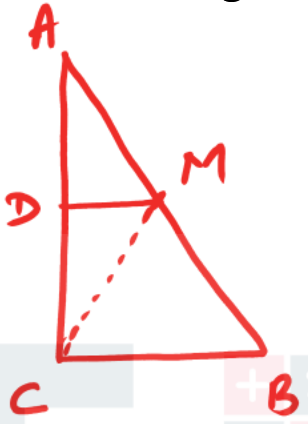
Questions

6. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that :

(i) D is the mid-point of AC

(ii) MD $\perp$ AC

(iii) CM = MA = AB/2



Given: $\angle C = 90^\circ$

M is mid point of AB.

$MD \parallel BC$

To prove:

Proof: In $\triangle ABC$,

M is mid point of AB

$MD \parallel BC$

$\therefore$ by converse of mid point theorem

D is mid point of AC.

$\angle MDA = \angle BCA$ (corresponding $\angle$s)

$\therefore \angle MDA = 90^\circ$

or $\boxed{MD \perp AC}$

In $\triangle ADM$ and $\triangle CDM$

$AD = CD$ ($\because$ D is mid point of AC)

$\angle ADM = \angle CDM$ (each 90°)

$DM = DM$ (common)

$\therefore \triangle ADM \cong \triangle CDM$ by SAS rule

$\therefore CM = AM$ (cpct)

$AM = \frac{1}{2} AB$ ($\because$ M is mid point of AB

$\therefore \boxed{CM = AM = \frac{1}{2} AB}$